The Simplest Parkour Model: Experimental Validation and Stability Analysis

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We describe and experimentally validate the Simplest Parkour Model (SPM) for the ParkourBot, a planar dynamic climbing robot equipped with two springy BowLegs. By controlling the leg angles and injected energy at impact, the ParkourBot is capable of climbing up and down in a rigid chute on an inclined air table. The SPM consists of a point mass and two massless legs. The legs are assumed to be infinitely stiff, resulting in an instantaneous stance phase and a closed-form solution of the hybrid dynamics. In this paper, we show that the SPM is a good predictor of the actual experimental behavior. Using the SPM we compute the fixed points, stability and basins of attraction of period-1 limit cycles.

1. Introduction

Parkour is a form of dynamic locomotion that exploits body dynamics and reaction forces from the environment to efficiently access places that are otherwise difficult to reach.\(^1\) Inspired by the flexibility of parkour, Carnegie

![Image of ParkourBot](a)

![Cartoon of ParkourBot](b)

Fig. 1. (a) Image of ParkourBot, a planar biped climbing robot, on an inclined air table to reduce gravity. (b) A cartoon of a simplified model of the ParkourBot ascending a chute.
Mellon University, in collaboration with Northwestern University, developed the ParkourBot (see Fig. 1(a)), a dynamic climbing robot equipped with two springy BowLegs. Potential energy is stored in the legs during flight and is quickly converted to kinetic energy at impact. By controlling the leg angles and injected energy, our eventual goal is to control the ParkourBot to reliably move through a complex environment of footholds. Our first step is to control the ParkourBot to dynamically climb in a chute on an inclined air table with reduced gravity as shown in the cartoon in Fig. 1(b).

This paper describes and experimentally validates the Simplest Parkour Model (SPM), a simple model with closed-form hybrid dynamics. With the SPM, we compute fixed points, stability and basins of attraction (BOA) of period-1 limit cycles.

In Section 2 we place this work in context of previous work. Section 3 briefly summarizes the mechanical design of the ParkourBot. The SPM is described in Section 4 with experimental validation in Section 5. We conduct stability analysis of this model in Section 6.

2. Related Work

A common model for dynamic running and hopping legged robots is the spring-loaded inverted pendulum (SLIP). The SLIP model has been used mainly to model robots traversing horizontal terrain such as Raibert’s monoped, Brown and Zeglin’s BowLeg hopper and RHex. The non-linear differential equations of the finite-time stance phase for the SLIP model cannot be solved analytically, so limit cycles and stability properties are computed numerically. Recently, Degani et al. modeled the ParkourBot as a two-legged SLIP and numerically analyzed its stability properties. They observed theoretically and experimentally that climbing up at constant speed (with a larger energy injection at impact) is more stable with a larger BOA than climbing down.

By analogy to the simplest walking model of passive dynamic walking and the simplified one-dimensional hopping robot, we investigate a simpler version of the SLIP model, which arose from personal communication with Degani. We refer to this model as the Simplest Parkour Model (SPM). The SPM is a special case of the SLIP model in that it has an instantaneous stance phase, enabling us to derive a closed-form solution of the hybrid dynamics and explicit expressions for fixed points of period-1 limit cycles.

The analytical nature of the SPM is particularly useful for motion planning applications based on sequential composition of asymptotically stable movements, known as Lyapunov funneling. Estimates of BOA can be used
to stably compose distinct controllers, such as for sequential paddle-and-ball batting maneuvers. A set of asymptotically stable walking gaits has been used as motion primitives for a bipedal robot, where walking paths are planned as sequences of controllers from the set of primitives. The same procedure could be used with the analytical SPM to plan provably stable transitions between climbing velocities.

3. Mechanical Design of ParkourBot

The ParkourBot is equipped with two springy BowLegs which are low weight energy-efficient springs that operate similarly to an archer’s bow. During flight, a servo motor retracts a string attached to the foot, storing energy in the bow. At impact the string goes slack and the leg extends quickly, converting the stored potential energy to kinetic energy of the ParkourBot’s body. Thus a low-power servo motor can store energy in the leg during the relatively long flight phase, and much of this energy is recovered in a high power event at impact. Another servo motor is used to independently change the leg angle. By controlling the leg angle and the stored potential energy, the robot is capable of climbing up and down as well as bouncing in place. Note that the robot does not adhere to or grab the wall, but instead uses the springy leg to kick off the wall. The robot is also equipped with a gyrostabilizer to minimize rotation in the plane of the air table. With the gyrostabilizer and low weight legs, a reasonable simple model of the ParkourBot is a point mass with massless legs. For a more complete description of the mechanical design, see Degani et al.

4. Simplest Parkour Model

The SPM consists of a point mass $m$ and two massless legs of length $\ell$. The position of the point mass is $q = (y, z)$ as shown in Fig. 1(b). The springy legs are modeled as infinitely stiff, leading to an instantaneous stance phase. The inertial frame is centered between the two vertical walls separated by a distance $2d$. An impact occurs at $y = \pm a$, where $a = d - \ell \cos \theta$ and $\theta$ is the leg angle. At impact, kinetic energy $KE$ is governed by

$$KE^+ = \epsilon KE^- + \sigma,$$

where $\epsilon \in (0, 1)$ is a restitution coefficient representing energy loss, $\sigma$ is energy injected at impact and the superscripts $-$ and $+$ refer to just before and after impact. Positive $\sigma$ corresponds to converting the energy stored during the flight phase into kinetic energy where as negative $\sigma$ corresponds
to storing more energy in the spring at impact. The current ParkourBot is not capable of negative $\sigma$, but future designs may be based on this idea.

The continuous dynamics are ballistic motion given by $[\ddot{y}, \ddot{z}]^T = [0, -g]^T$. At impact, an impulse is applied along the leg through the mass. We represent the robot’s velocity at impact as $[v_n, v_r]^T = R(\theta - \pi/2)[\dot{y}, \dot{z}]^T$, where $R(\cdot)$ is the standard counterclockwise rotation matrix and $(v_n, v_r)$ represent the normal (perpendicular to the leg) and radial (along the leg) velocities, respectively, as shown in Fig. 1(b). Since the impulse can only influence the radial velocity, the normal velocity remains constant through the impact. The impact map of the SPM is

$$ q^+ = q^-, \quad v_n^+ = v_n^-, \quad v_r^+ = \sqrt{\epsilon(v_r^-)^2 + \frac{2\sigma}{m}}, $$

where the controls for the ParkourBot are $\xi = (\theta, \sigma)$.

We compose the integrable flight dynamics and the impact map (2) to find a closed-form solution from one post-impact state to the next. We do not write the solution here due to its length. Because the dynamics are invariant to the robot’s height $z$, and because $y$ at impact is determined by the leg angle $\theta$, we restrict our analysis to the post-impact velocities.

By symmetry, we can assume the robot is always impacting the right wall; the position and velocity are reflected to the left wall immediately after impact. The velocity update map is $x_{k+1}^+ = P(x_k^+, \xi)$, where $x^+ = [\dot{y}^+, \dot{z}^+]^T$ is the velocity vector immediately after an impact. If the post-impact velocity $\dot{y}_{k+1}^+$ is negative, then the impulse did not reverse the sign of the horizontal velocity, and the point mass continues toward impact with the wall. We call this failure mode “tripping,” resulting in the robot falling. We can solve analytically for the region of the $(\dot{y}^+, \dot{z}^+)$ space that leads to tripping on the next impact.

5. Experimental Validation of the SPM

Using a high-speed PhotonFocus TrackCam mounted above the air table, we recorded the positions of two LEDs fixed on the robot at a rate of 500 Hz to compute the position and orientation of the robot’s center of mass. We recorded the trajectory through impact for many impacts with various leg retract controls and fixed leg angles of $\theta = \pi/6$. The leg retract controls correspond to different injected energies $\sigma$.

A recording for a single impact is shown in Fig. 2(a). As shown in Fig. 2(b), the body angle stays approximately constant throughout the trajectory due to the gyrostabilizer, allowing us to ignore orientation in
Fig. 2. (a) Recorded trajectory of ParkourBot for one impact. (b), (c), and (d) show the body angle, horizontal position $y$ and vertical position $z$ as a function of time, respectively, of the ParkourBot for one impact. (c) and (d) have linear and quadratic regressions overlaid for pre-impact and post-impact flights.

the SPM. During flight, the horizontal and vertical positions respectively follow linear and quadratic trajectories as predicted by ballistic motion, shown in Fig. 2(c) and Fig. 2(d). We fit linear and quadratic regressions to the pre- and post-flight positions. The impact is estimated to occur at the intersection of the linear regressions. With this method, we do not estimate the robot’s state during the stance phase and focus on finding an instantaneous impact state that is compatible with the two flight phases. With the regressions, we estimate the pre- and post-impact states for the SPM. To compute the normal and radial velocities, we calculate the effective angle $\theta$ with the wall by combining the angles of the leg and the body.

The kinetic energy and normal velocity for post- versus pre-impact for three sample energy controls for many impacts can be seen in Fig. 3(a) and 3(b), respectively. The slopes of linear regressions of the energy data were averaged over many controls to produce $\epsilon = 0.7$, which is comparable to energy loss measured with the single leg BowLeg hopper. The energy intercept gives an approximately linear relationship between the leg retract
control and the injected energy. Other parameters were measured to be \( a = 0.1 \text{ m}, \ m = 1.5 \text{ kg}, \ \text{and} \ g = 0.98 \text{ m/s}^2 \). These experiments show that the SPM can be used to approximately model the ParkourBot.

6. Fixed Points and Stability Analysis

A fixed point \( x_\ast \) (corresponding to a period-1 limit cycle) satisfies \( x_\ast = P(x_\ast, \xi) \). The velocities of the period-1 fixed points can be computed as

\[
\begin{bmatrix}
\dot{y}_\ast^+ \\
\dot{z}_\ast^+
\end{bmatrix} = \left[ \tan \theta - \frac{2}{(1-\epsilon) \sin 2\theta} \left( 1 + \epsilon - \sqrt{4\epsilon + (1-\epsilon) \frac{\sigma}{m ga} \sin 2\theta} \right) \right] \sqrt{ag \cot \theta}.
\]

The change in height per bounce \( h \) for the fixed point is given by

\[
h = - \frac{4a}{(1-\epsilon) \sin 2\theta} \left( 1 + \epsilon - \sqrt{4\epsilon + (1-\epsilon) \frac{\sigma}{m ga} \sin 2\theta} \right).
\]
Fig. 4. (a) Maximum eigenvalue $||\lambda_{\text{max}}||$ of the linearized discrete dynamics as a function of energy control $\sigma$ for fixed leg angle $\theta = \pi/6$. (b) - (d) BOA in the velocity $(\dot{y}, \dot{z})$ space for three controls with their respective fixed points. The immediate trip region corresponds to points that do not reverse the horizontal velocity after impact.

We can solve (4) for the control $\sigma$ as

$$\sigma = mg \left( \frac{(1 - \epsilon)}{\sin(2\theta)} + \frac{(1 + \epsilon)h}{2a} + \frac{(1 - \epsilon)h^2 \sin 2\theta}{16a^2} \right),$$

which allows us to re-parameterize the fixed point with $h$ as

$$\dot{y}^*_* = \sqrt{ag \cot \theta}, \quad \dot{z}^*_* = \left( \tan \theta + \frac{h}{2a} \right) \sqrt{ag \cot \theta}.$$

With this parameterization, we can more easily create motion plans than with the numerical results of the SLIP model.

If the maximum eigenvalue of $\partial P/\partial x$, evaluated at the fixed point $x_*$, has magnitude less than one, the system is asymptotically stable. Using the calibrated parameters and a leg angle of $\theta = \pi/6$, we computed the maximum eigenvalue as shown in Fig. 4(a), which shows climbing up is more stable than descending.

For asymptotically stable fixed points, the BOA is the set of points that will converge to the fixed point as the number of impacts approaches $\infty$. We estimate the BOA by simulating a sample grid in the velocity space and
determining which points converge to the fixed point. Fig. 4(b) - 4(d) show the fixed point, the BOA and an immediate trip region for energy controls $\sigma = -0.03$, $0$, and $0.05$ J and fixed leg angle $\theta = \pi/6$. The immediate trip region is shown in red. The intermediate white space contains points that will eventually trip. The area of the BOA for the SPM decreases with decreasing energy levels, which was also observed with the SLIP model.

7. Conclusion

We describe the Simplest Parkour Model, experimentally validate the model and analyze the stability and basin of attraction of period-1 fixed points. The stability and BOA of the SPM qualitatively match those of the SLIP model. An advantage of the SPM is the closed-form hybrid dynamics, allowing for analytical expressions of the fixed points. The analytical nature of the fixed points can more easily be utilized in motion plans.

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References