Optimal Sampling of Recruitment Curves for Functional Electrical Stimulation Control

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***Abstract*— A major challenge in controlling multiple-input multiple output functional electrical stimulation systems is the large amount of time required to identify a workable system model due to the high dimensionality of the space of inputs. To address this challenge we are exploring optimal methods to sample the input space. In this paper we present two methods for optimally sampling isometric muscle force recruitment curves. One method maximizes the information about the recruitment curve parameters, and the second method minimizes the average variance of the predicted output force. We compared these methods to two previously-used methods in simulation. The simulation model was identified from recruitment data collected during experiments with a human subject with a high spinal cord injury. The optimal sampling methods on average produced estimates of the output force with less error than the two previously-used methods. The optimal sampling methods require fewer system identification experiments to identify models with similar output prediction accuracy.**

I. INTRODUCTION

Functional electrical stimulation (FES) is a strategy to restore lost functions to persons with paralysis. Many tasks, such of reaching motions of the arm, require using a continuous range of muscle force. A model of the relationship between the stimulation inputs to the muscle and the resulting force output of the muscle is critical in designing controllers for these tasks. The isometric recruitment curve is often used to represent the stimulation input/force output relationship and is valuable in FES controller design.

Methods for identifying the recruitment curve have been previously studied [1]. The simplest and most used method is the steady-state step response method [1]. For the steady-state step response method, a constant input, usually a stimulation amplitude or pulse width, is applied, and the output is averaged after it reaches steady state. Inputs are selected evenly over the domain of possible inputs. When ample experiment time exists the recruitment curve can be sampled at many input levels several times each to estimate the mean and variance of the force output at each input level.

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Densely sampling the input space for FES tasks requiring coordination of multiple muscles is infeasible. Experiments with FES in human subjects are often limited by time and human fatigue. Multi-muscle tasks often require identifying recruitment curves, muscle dynamics, and skeletal dynamics. Each addition of complexity adds extra dimensions to the input and state space of the FES system to explore. To save time and effort in experiments and to decrease errors in control tasks we are examining optimal methods to sample recruitment curves as an alternative to dense sampling.

Optimal sampling methods are well-studied for functions that are similar to the sigmoid functions used to model recruitment curves [2]. These methods are often called optimal experiment designs. Some examples include psychometric functions in two-alternative forced-choice experiments, dose-response curves for medicines, and reliability curves in engineering.

In this paper we apply two well-known methods from optimal experiment design [3] to the problem of sampling recruitment curves and compare them to two previously-used sampling methods. We compare the methods in a simulation based on recruitment curves identified for a human subject with a high spinal cord injury. The first optimal method minimizes the variance of the parameters of the recruitment curve model by choosing inputs that maximize information about the parameters. The second optimal method minimizes the average variance of the predicted outputs of the recruitment curve model. The first previously-used method evenly samples the input domain. The second previously-used method [4], bisects adjacent pairs of previous inputs. The next input is the mean of the pair of adjacent inputs with the largest difference in output.

To our knowledge, this is the first investigation of optimal experiment designs for the sampling of muscle recruitment curves. Because we are motivated by limited experiment time for identifying human FES systems, the goal of this study is to determine if the optimal sampling methods produce recruitment curve estimates with less output prediction error and variance than those produced with previously-used sampling methods for small numbers of experiments.

II. METHODS

A. Recruitment Curve Model

We model the recruitment curve with a sigmoid function

$$f = \frac{a}{1+e^{b(c-u)}} - \frac{a}{1+e^{bc}}, \tag{1}$$

where u is the stimulation input, typically a stimulation pulse width or amplitude, f is the force output of the muscle,

a is the maximum output of the sigmoid function, b is proportional to the slope of the sigmoid function at 50% of the maximum output, and c is the input at which the sigmoid function outputs 50% of its maximum output. The second term on the right-hand side is an offset term that forces the output to be zero when the input is zero.

The goal is to identify the parameters in (1) by doing experiments with a simulated FES system. A single data point consists of applying one input to the system and measuring the output. A series of data points defines an experiment design. The size of an experiment design is the number of data points making up the design. By sampling the domain of the input u, and measuring the resulting output f, we estimate the parameters, a, b, and c using maximum likelihood estimation (MLE). Having computed the best estimate of the parameters, we also estimate the force output $f(u)$, and its variance $\sigma_f^2(u)$, at any input u.

B. Sampling Methods

We use four experiment design methods for sampling the inputs to estimate the parameters in (1). The first two methods, which we call even and bisection, do not explicitly optimize anything. The third and fourth methods, called D-optimal and V-optimal in optimal experiment design literature [3], explicitly minimize objective functions.

The even method samples the recruitment curve at evenly-spaced levels in the domain of inputs. The entire even experiment design is determined before any experiment design sequence begins. We compute the MLE of the parameters after all data points of an even design are collected.

The bisection method [4] first gathers three data points: one at a low input level, one at a middle input level, and one at a high input level. The difference in force output between the low and middle inputs and the difference in force output between middle and high inputs are compared. The pair of adjacent inputs with the largest output difference is bisected, meaning that the mean of those inputs becomes the input for the next experiment. Each subsequent input is the mean of the adjacent previous inputs that have the largest difference in muscle force output. We compute the MLE of the parameters after all data points in a bisection design are collected.

The D-optimal method determines the input of the next data point or vector of inputs for the next series of data points by solving

$$\mathbf{u}^* = \underset{\tilde{\mathbf{u}}}{\operatorname{argmin}} - \log |M(\tilde{\mathbf{u}}, \mathbb{U})|, \quad (2)$$

where $M \in \mathbb{R}^{3\times3}$ is the information matrix that depends both on all previous inputs $\mathbb{U}$, and the next candidate input or vector of inputs $\tilde{\mathbf{u}}$, which is the variable over which to optimize. The information matrix is computed by

$$M = F^{\mathrm{T}} F, \quad (3)$$

where F is the parameter sensitivity matrix

$$F = \begin{bmatrix} \frac{df}{da}(u_1) & \frac{df}{da}(u_2) & & \frac{df}{da}(u_n) & \frac{df}{da}(\tilde{u}) \\ \frac{df}{db}(u_1) & \frac{df}{db}(u_2) & \cdots & \frac{df}{db}(u_n) & \frac{df}{db}(\tilde{u}) \\ \frac{df}{dc}(u_1) & \frac{df}{dc}(u_2) & & \frac{df}{dc}(u_n) & \frac{df}{dc}(\tilde{u}) \end{bmatrix}, \quad (4)$$

and u_i is the input of the i^{th} data point of the n total previous data points. This method is called D-optimal because it maximizes the *determinant* of the information matrix by selecting the inputs where the parameters are most sensitive. It is equivalent to minimizing the variance/covariance ellipsoid of the model parameters.

The V-optimal method determines the next input or vector of inputs by minimizing the predicted *variance* of the output integrated over the entire domain of the stimulation input,

$$\mathbf{u}^* = \underset{\tilde{\mathbf{u}}}{\operatorname{argmin}} \int_U \sigma_f^2(u, \tilde{\mathbf{u}}, \mathbb{U}) du. \quad (5)$$

The predicted variance σ_f^2, is

$$\sigma_f^2(u, \tilde{\mathbf{u}}, \mathbb{U}) = F_1^{\mathrm{T}}(u) M^{-1}(\tilde{\mathbf{u}}, \mathbb{U}) F_1^{\mathrm{T}}(u), \quad (6)$$

where $F_1(u)$ is a vector representing F for a single input u.

Implementing the D-optimal and V-optimal designs requires prior knowledge of the model parameters. Because (1) is nonlinear in the parameters, its derivatives, which show up in the objective functions to be minimized, are functions of the parameters themselves.

To determine the importance of having good prior estimates of the model parameters in computing optimal designs we compare estimates of recruitment curves using two procedures to find the initial four inputs required to solve for the parameters of (1) and estimate the variance of the output. The first initialization procedure assumes perfect knowledge of the real parameters by using the parameters of the simulation model. Using these parameters we solve (2) in the D-optimal method and (5) in the V-optimal method to find the first four inputs. The second procedure assumes no knowledge of the parameters and uses the bisection method to determine the inputs for the first four data points.

Both the D-optimal and V-optimal designs are sequentially computed. After collecting the first four data points we compute the MLE of the parameters and use these new parameters to solve (2) or (5) for the next input, collect the next data points, and compute new parameter estimates. The collections of all subsequent data points proceeds similarly.

C. Evaluating the Sampling Methods

To test the relative ability of the four experiment design methods to reduce both the error and variance of the output predictions of the identified models we use a simulation model,

$$f = \frac{a}{1 + e^{b(c-u)}} - \frac{a}{1 + e^{bc}} + N(0, \sigma^2). \quad (7)$$

The simulation model was identified previously for nine different electrically stimulated muscle units of a human subject who sustained a hemisection of the spinal cord at the C1-C2 level [5]. The subject has surgically implanted electrodes that can stimulate various nerves and muscles used to move the arm. We refer to a muscle unit as either an individual muscle or a group of muscles that contract when a specific nerve is stimulated. Note that the only difference between (1) and (7) is the addition of normally-distributed noise with variance σ^2 in (7). The model assumes that the noise does not vary with stimulation level for electrically stimulated muscles [6]. The values of the parameters of (7) varied across the nine muscle units.

Using each of the four experiment design methods we determined a sequence of stimulation inputs and randomly drew force outputs from (7) given the inputs. We ran experiment designs of increasing size, starting with four data points and ending with twelve data points per experiment design. For each muscle unit we used the four experiment design methods 500 times for each size experiment design.

The four experiment design methods are scored as follows. We treat the mean output of the simulation model at each stimulation input as the ground truth. The bias, defined as the difference between the output predicted by the identified model and the mean output of the simulation model, is computed at 1% increments. For a given trial the RMS bias over all the stimulation inputs is computed. The variance is computed at the 1% stimulation level increments and averaged. We take the square root of the average variance. For each size experiment design and each experiment design method there were 500 measures of the RMS bias and square root of the average variance, one for each random trial. We use the median of the 500 measures to score each design method for each experiment design size. The median is used because the variance can be extremely high for some trials with low experiment design size. With large outliers, the median represents the central tendency better than the mean.

The MATLAB® function `fmincon()` was used to solve for the MLE of the parameters of (1) and the D-optimal and V-optimal experiment inputs as in (2) and (5).

III. RESULTS

When accurate prior knowledge of the parameters of the recruitment curve model was assumed, the D-optimal and V-optimal experiment designs yielded recruitment curve estimates with less bias than did the even and bisection experiment designs (Fig. 1). For seven of the nine muscle units the D-optimal and V-optimal designs produced less-biased estimates for every experiment design size. For the other two muscle units either the bisection or even design performed as well in terms of bias as the D-optimal or V-optimal for larger experiment designs.

When no prior knowledge of the parameters was assumed all methods yielded similar bias for small experiment designs, but the D-optimal and V-optimal methods yielded lower bias than the even and bisection methods for designs with more experiments (Fig. 1). With no knowledge of the real parameters, the first several experiments were suboptimal for all methods. With new experiments the information about the parameters increased, and the subsequent D-

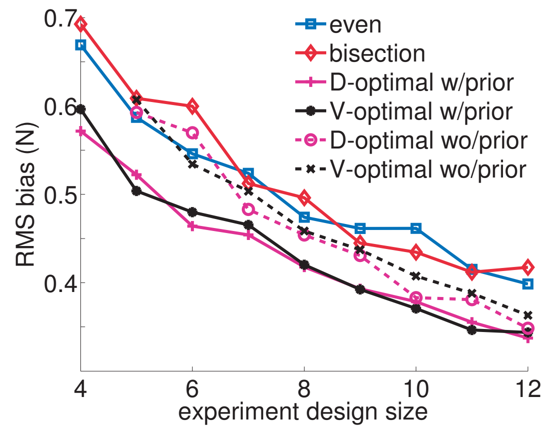

Fig. 1. RMS bias vs. experiment design size for the simulated radial nerve muscle unit. The D-optimal and V-optimal methods are displayed both when prior knowledge of the parameters was available (w/prior) and when no prior knowledge was available (wo/prior). Markers for a size four experiment design are not included for D-optimal wo/prior and V-optimal wo/prior because they are identical to the bisection method.

optimal and V-optimal experiments were optimal for the updated parameter estimates and yielded less-biased estimates of the muscle force output.

The four experiment designs resulted in different sampling of the input space. The even design samples the input space evenly (Fig. 2(a)). The bisection design attempts to sample the output space evenly (Fig. 2(b)). The even and bisection designs run one experiment at several unique input levels. The D-optimal and V-optimal designs run multiple experiments in clusters near the three inputs where there is the most information about the parameters of the recruitment curve model, which is where the derivatives of the parameters are highest (Fig. 2(c)-(d)). The information matrix M, which is part of both the D-optimal and V-optimal objective functions, includes the parameter derivatives. The derivatives are highest at 100% stimulation for the magnitude parameter a, and at just below and above 50% of the maximum output for the combination of the slope parameter b, and the 50% of maximum output parameter c. Note that the sequential experiment design updates the estimate of the parameters and the optimal experiment after each experiment. The result is designs with three clusters of inputs rather than three inputs exactly repeated multiple times.

Repeatedly placing inputs at the most sensitive stimulation levels, as the D-optimal and V-optimal designs do, reduces output bias at those inputs, as the data approaches the true mean with more and more data points. An example of this is at the 100% input level in Fig. 2. When only one noisy point was sampled at 100% input, as with bisection (Fig. 2(b)), there was bias in the estimate of the output at 100% input. When multiple noisy points were sampled at 100% input, as with the D-optimal and V-optimal designs, there was less bias in the estimate of the output at 100% input.

Taking just one sample at a particular input, as the even and bisection methods do, leads to a biased, one-point estimate of the output for that input. Bias is magnified when the outlying data point is at an input where the parameters of

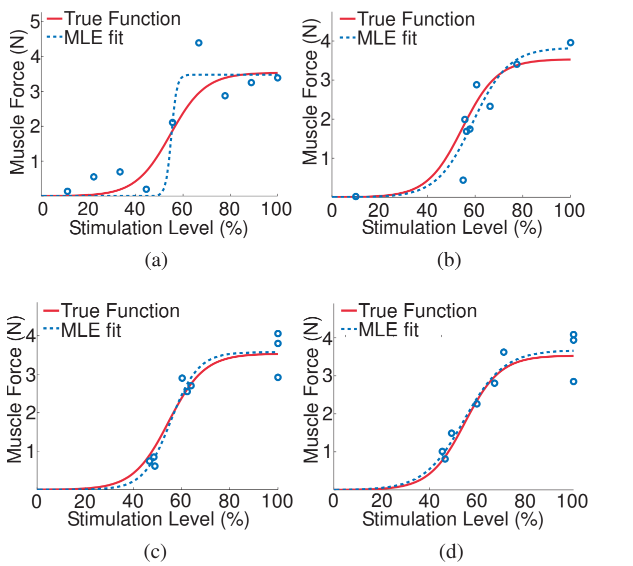

Fig. 2. Example experiment designs for the (a) even, (b) bisection, (c) D-optimal, and (d) V-optimal design methods.

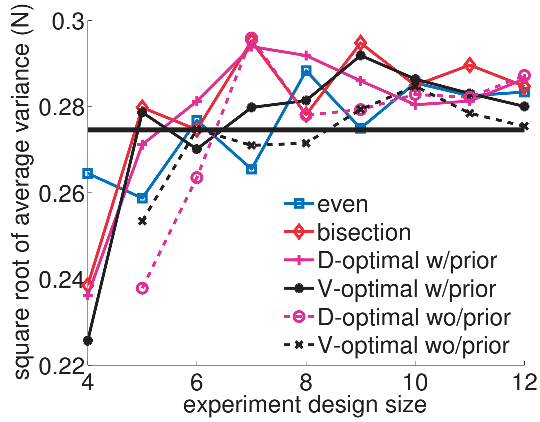

Fig. 3. Square root of the average variance vs. experiment design size for the simulated suprascapular nerve muscle unit. The solid horizontal black line is the square root of the variance of the simulation model. Markers for size four experiment designs are not included for D-optimal wo/prior and V-optimal wo/prior because they are identical to the bisection method.

the model are most sensitive. An example is seen in Fig. 2(a) where the outputs at just over 40% and just over 60% input level are far from the true mean. These two points caused a particularly biased estimate of the recruitment curve.

There was no evidence to suggest that any of the four design methods was superior in reducing average variance (Fig. 3). This was true for all nine simulated muscle units. The output variance of the simulation model does not vary with input, so only a few experiments are required to estimate the average variance no matter what experiment design method is used. One would expect that the V-optimal design, which minimizes the integral of the output variance, would be superior in reducing average variance. We used a V-optimal design that minimized variance over the entire input space rather than in a particular area of input space. When changing the V-optimal cost function to minimize variance in a specific range of inputs, the estimate of the output variance in this range was least for the V-optimal design and greater for the other designs.

IV. DISCUSSION

This study aimed to determine if using optimal sampling methods decreases bias and variance in estimates of isometric muscle force recruitment curves. By simulating experiments with four different design methods 500 times for each experiment design size we showed that the bias in estimating recruitment curves was smallest for the optimal design methods. The median size of the output variance over the 500 trials did not depend on the design method.

While our optimal designs were superior in reducing bias to the standard evenly-spaced design and the previously used bisection method [4], the benefit of using the optimal experiment designs, averaged over many simulation trials, was on the order of 0.1 N (Fig. 1) which is small compared to an average standard deviation of approximately 0.3 N. It must also be mentioned that the bias of a particular recruitment curve estimate given a small set of data points has more to do with the particular data points rather than the method used in obtaining them.

Still there are important reasons to use the optimal designs. The first is that they protect against particularly "bad" outliers, which are most damaging to predictions used for control when the data set is small. This can be seen from the outliers that caused a biased fit with the even design in Fig. 2(a). So while on average the optimal methods might not reduce bias very much, they can reduce bias significantly in the worst case of outliers.

A second reason to further explore optimal designs for modeling of FES systems for control is that many FES tasks require multiple muscles acting over multiple joints. The input space is much larger as the task becomes more complicated. We show in this paper that optimal designs decrease bias in predictions of the output when the input space is one-dimensional. When exploring higher and higher-dimensional input spaces optimal designs will become more and more important both in reducing bias and variance.

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